RESEARCH ARTICLE

Fuzzy $g^{\#\#}$- Connectedness in Fuzzy Topological Spaces

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ABSTRACT: In this paper, we introduced new results in fuzzy connected spaces, that is $g^{\#\#}$-connectedness in fts by using $g^{\#\#}$-open sets, $g^{\#\#}$-closed sets in fts. We studied some of their properties and also investigated their characterization. Further this work lead to the study of disconnectedness of fuzzy $g^{\#\#}$-closed sets.

KEYWORDS: Fuzzy $g^{\#\#}$-closed set, Fuzzy $g^{\#\#}$-Connectedness, Fuzzy $g^{\#\#}$-separated, Fuzzy super $g^{\#\#}$-connectedness, Fuzzy strongly $g^{\#\#}$-connectedness

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1. INTRODUCTION

L.A. Zadeh [1] introduced the concept of 'fuzzy subset' in 1965. Fuzzy subset provides a natural framework for generalizing concepts of general topology into the Fuzzy topological spaces. Levine [2] introduced generalized closed sets and strongly connected sets in general topology in 1965. Dense topology was introduced in 1968. The D space is said to be a topological space, if every non empty open set is dense in it. A super connected space is said to be a topological space, if it has no proper regular open subset and the space is super connected in D space. In 1968, C.L. Chang [3] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Which includes a open set, closed set, neighborhood, interior set, continuity and compactness and connectedness. Hutton [4] in 1975 introduced normality in fuzzy topological spaces. The space which has no proper fuzzy clopen set is fuzzy connected space; otherwise it is fuzzy disconnected space. Pu and Liu [5] in 1980 defined connectedness by using the concept of the fuzzy closed set. Lowen [6] in 1981, defined an extension of connectedness in a particular family of fuzzy topology. Zheng [7] in 1984, introduced fuzzy path and fuzzy connectedness. In 1985, Fatteh and Bassam[8] studied further the notation of fuzzy super connected and fuzzy strongly connected spaces for a crisp set in a fuzzy topological spaces. In 1991, Ajmal and Kohli[9] extended the notion of connectedness to an arbitrary fuzzy set, c-zero dimensional , strongly zero connected , total disconnected and Local connectedness in fuzzy topological spaces. In 2004, Balasubramanian and Chandrasekhar [10] studied connected and disconnectedness by using α-open sets in fuzzy topological spaces. Hassan [11] in 2007, studied some kind of fuzzy connected spaces.

In this paper, by using $g^{\#\#}$-closed and $g^{\#\#}$- open fuzzy set, which are obtained by generalization via α open set [12,13]. we introduced few more results as an extension of a connected spaces like, Fuzzy $g^{\#\#}$-Connectedness, Fuzzy super $g^{\#\#}$ -connectedness, Fuzzy strongly $g^{\#\#}$-connectedness .

2. PRELIMINARIES

In this paper we shall denote a fuzzy topological spaces by (X,T) and (Y, σ) where X and Y are sets and T and σ is are fuzzy topologies. The notations $\delta, \gamma, \mu, \alpha, \beta, u, v$ are ued to denote fuzzy sets in fts.

Definition 2.01: A fuzzy set (X,T) is said to be connected, if it has no proper fuzzy clopen set. Otherwise it is called fuzzy disconnected. [4]

Definition2.02: A fuzzy set λ in fts (X,T) is proper if $\lambda \neq 0$ and $\lambda \neq 1$.[4]

Definition 2.03: A fts (X,T) is called fuzzy super connected if it has no proper fuzzy regular open set. [8]

Definition2.04: An fts (X,T) is called fuzzy strongly connected, if it has no non zero fuzzy closed sets f and k such that $f + k \leq 1$. If X is not fuzzy, strongly connected then it will be called as fuzzy weakly disconnected. [8]

Definition 2.05: A fuzzy set (X,T) is said to be fuzzy α-connected, if (X,T) has no proper fuzzy set λ which is both fuzzy α-open and fuzzy α-closed. [10]

Definition 2.06: A fuzzy λ in a fts (X,T) is called fuzzy regular α-open set, if $\lambda = (\underline{\lambda})_0$.[10]

Definition 2.07: A fuzzy topological spaces X is said to be a T_1-fuzzy topological space, if every fuzzy point in X is fuzzy closed.[5]

Definition 2.08: A fuzzy topological space X is aid to be fuzzy locally connected at a fuzzy point x_α in X , if for every fuzzy open set μ in X containing x_α, there exist a connected fuzzy open set δ in X such that $x_1 \leq \delta \leq \mu$. [9]

3. FUZZY $g^{\#\#}$- CONNECTEDNESS

Definition 3.01: A fts (X,T) is said to be fuzzy $g^{\#\#}$-connected (briefly $f g^{\#\#}$-connected), if fts (X,T) has no proper fuzzy $g^{\#\#}$-open and $g^{\#\#}$- closed sets, where a fuzzy set δ in (X,T) is proper, if $\delta \neq 0$ and $\delta \neq 1$.

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Theorem 3.02 : A fts (X,T) is fuzzy $g^{\#\#}$-connected, iff it has no non zero fuzzy $g^{\#\#}$-open sets δ_1 and δ_2 such that $\delta_1 + \delta_2 = 1$.

Proof: If δ_1 and δ_2 exist, then δ_1 is a proper fuzzy $g^{\#\#}$- closed and $g^{\#\#}$-open set in (X,T). If fts (X,T) is not fuzzy $g^{\#\#}$-connected. Then it has a proper fuzzy set δ_1 which is fuzzy $g^{\#\#}$-open set and also a fuzzy $g^{\#\#}$- closed set. Assume $\delta_2 = 1 - \delta_1$ is a fuzzy $g^{\#\#}$-open set in X. Then δ_2 is a fuzzy $g^{\#\#}$-open set, such that $\delta_2 \neq 0$ and $\delta_1 + \delta_2 = 1$.

Corollary 3.03: A fts (X,T) is fuzzy $g^{\#\#}$-connected, iff it has no non zero fuzzy sets δ_1 and δ_2 such that $\delta_1 + \delta_2 = 1$, $\overline{\delta_1} + \delta_2 = \delta_1 + \overline{\delta_2} = 1$.

Definition 3.04: If $A < X$, X is an fts, then A is said to be fuzzy $g^{\#\#}$-connected subset of (X,T), if A is a fuzzy $g^{\#\#}$-connected space a fuzzy subspace of (X,T).

If $A < Y < X$, then A is a fuzzy $g^{\#\#}$-connected subset of (X,T), iff it is a fuzzy $g^{\#\#}$-connected subset of the fuzzy subspace Y of X.

Theorem 3.05: If (X,T) is fts and Let A is a fuzzy $g^{\#\#}$-connected subset of (X,T). Let δ_1 and δ_2 are non zero fuzzy $g^{\#\#}$-open sets in fts (X,T) with $\delta_1 + \delta_2 = 1$, then either $\frac{\delta_1}{A} = 1$ or $\frac{\delta_2}{A} = 1$

Proof: If there exist $x, y \in A$ such that $\delta_1(x) \neq 1$ and $\delta_2(y) \neq 1$. Then $\delta_1 + \delta_2 = 1$, which implies that $\frac{\delta_1}{A} + \frac{\delta_2}{A} = 1$, where $\frac{\delta_1}{A} \neq 0$ and $\frac{\delta_2}{A} \neq 0$. Therefore δ_1 is a proper $g^{\#\#}$- closed and $g^{\#\#}$-open fuzzy set in (X,T). $\delta_2 = 1 - \delta_1$ is a fuzzy $g^{\#\#}$- if open set such that $\delta_2 \neq 0$ and $\delta_1 + \delta_2 = 1$. Hence A is not a fuzzy $g^{\#\#}$-connected space a fuzzy subspace of (X,T).

Definition 3.06: Let (X,T) be fts, two non empty fuzzy subsets δ_1 and δ_2 in fts (X,T) are said to be fuzzy $g^{\#\#}$-separated (briefly $fg^{\#\#}$-separated), if $cl(\delta_1) + \delta_2 \leq 1$ and $\delta_1 + cl(\delta_2) \leq 1$ where $\delta_1 \wedge cl(\delta_2) = 0$ and $cl(\delta_1) \wedge \delta_2 = 0$. That is $[\delta_1 \wedge cl(\delta_2)] \vee [cl(\delta_1) \wedge \delta_2] = 0$.

Theorem 3.07: Let $\{A_\alpha\}_{\alpha \epsilon \wedge}$ be a family of fuzzy $g^{\#\#}$-connected subset of (X,T), such that for each and β in ∧ and $\alpha \neq \beta$, μ_{A_α} and μ_{A_β} are not $g^{\#\#}$-separated from each other. Then $\vee_{\alpha\epsilon\wedge}\{A_\alpha\}$ is a fuzzy $g^{\#\#}$-connected subset of (X,T).

Proof: Proof Omitted

Corollary 3.08: If $\{A_\alpha\}_{\alpha\epsilon\wedge}$ is a sequence of a fuzzy $g^{\#\#}$-connected subset of (X,T). Let $\wedge_{\alpha\epsilon\wedge}\{A_\alpha\} \in \emptyset$. Then $\vee_{\alpha\epsilon\wedge}\{A_\alpha\}$ a fuzzy $g^{\#\#}$-connected subset of (X,T).

Corollary 3.09: If $\{A_i : i = 1,2,3,4, \ldots.\}$ is a sequence of a fuzzy $g^{\#\#}$-connected subset of (X,T), such that μ_{A_i} and $\mu_{A_{i+1}}$ are not separated from each other for $i = 1,2,3,4 \ldots.$ Then $\vee_{i=1}^{\infty}\{A_i\}$ is a fuzzy $g^{\#\#}$-connected subset of (X,T).

Theorem 3.10: If A and B are subsets of an fts (X,T). Let $\mu_A \leq \mu_B \leq cl(\mu_A)$ and let A is a fuzzy $g^{\#\#}$-connected subset of X, then B is also an a fuzzy $g^{\#\#}$-connected subset of (X,T).

Proof: Proof omitted

4. FUZZY SUPER $g^{\#\#}$-CONNECTEDNESS IN FTS

Definition 4.01: A fuzzy set δ in a fts (X,T) is said to be fuzzy $g^{\#\#}$- regular open set, if $\delta = cl(int\ \delta)$.

Definition 4.02 : A fts (X,T) is said to be Fuzzy Super $g^{\#\#}$-connected, if there is no proper fuzzy $g^{\#\#}$- regular open sets δ_1 and δ_2 such that $\delta_1 + \delta_2 \leq 1$.

Theorem 4.03: If (X,T) is an fts then the following are equivalents

1) (X,T) is fuzzy Super $g^{\#\#}$-connected
2) Closure of every non zero fuzzy $g^{\#\#}$-open set in (X,T) is 1, that is $cl(\delta) = 1$.
3) Interior of every fuzzy $g^{\#\#}$-closed set in (X,T) is different from 1, that is zero ($\delta \neq 1$), that is $int(\delta) = 0$.
4) (X,T) does not have non zero fuzzy $g^{\#\#}$-open sets δ_1 and δ_2 such that $\delta_1 + \delta_2 \leq 1$.
5) (X,T) does not have non zero fuzzy sets δ_1 and δ_2 such that $cl(\delta_1) + \delta_2 = \delta_1 + cl(\delta_2) = 1$.
6) (X,T) does not have non zero fuzzy $g^{\#\#}$-closed sets λ_1 and λ_2 such that $\lambda_1 + \lambda_2 \leq 1$.

Proof: Proof omitted

Theorem 4.04: If (X, T) and (Y, σ) are fts . Let a function $f: X \to Y$ is fuzzy continuous, then (X,T) is fuzzy super $g^{\#\#}$ -connected.It implies that Y is fuzzy super $g^{\#\#}$-connected.

Proof: Proof omitted

Definition 4.05: A subset of an fts (X,T) is said to be fuzzy super $g^{\#\#}$-connected subset of (X,T), if it is fuzzy super $g^{\#\#}$-connected fts a a fuzzy subspace of (X,T).

Definition 4.06: Let (X,T) be a fts and If $A < Y < X$, then A is a fuzzy super $g^{\#\#}$-connected subset of (X,T), iff it is a fuzzy super $g^{\#\#}$-connected subset of the fuzzy subspace Y of (X,T).

Theorem 4.07: Let A be a fuzzy super $g^{\#\#}$-connected subset of an fts (X,T), then there exist fuzzy $g^{\#\#}$-closed sets f and g in (X,T) such that $int(f) + g = f + int(g) = 1$, then $\frac{f}{A} = 1$ or $\frac{g}{A} = 1$.

Proof: If $f(x_0) \neq 1$, $g(y_0) \neq 1$ and $x_0, y_0 \in A$ then $int[f(y_0)] + g[y_0] = 1$ and $f(x_0) + int[g(x_0)] = 1$ implies that $int[f(y_0) \neq 0$ and $int[g(x_0)] \neq 0$.Thus $\frac{int(f)}{A}$ and $\frac{int(g)}{A}$ are non zero fuzzy $g^{\#\#}$-open sets in A such that $\frac{int(f)}{A} + \frac{int(g)}{A} \leq 1$, which contradicts the fact that A I a fuzzy super $g^{\#\#}$- connected subset of (X,T).

Theorem 4.08: Let (X,T) be a fts and $A < X$ be a fuzzy super $g^{\#\#}$- connected subset of (X,T) such that μ_A is $g^{\#\#}$- open set of (X,T). If δ is a fuzzy regular $g^{\#\#}$- open set of (X,T), then either $\mu_A \leq \delta$ or $\mu_A \leq 1 - \delta$.

Proof: If $\delta = 0$ or $\delta = 1$, then the result holds good for $\lambda \neq 0$ and $\lambda \neq 1$. Let $f = cl(\delta)$ and $g = 1 - \delta$. Then f and g are such that $int(f) + g = f + int(g) = 1$. By hypothesis $\mu_A \leq f$ or $\mu_A \leq g$. So $\mu_A \leq int(f)$ or $\mu_A \leq int(g)$ as μ_A is fuzzy $g^{\#\#}$-open . Therefore $\mu_A \leq cl[int(\delta)] = \delta$ or $\mu_A \leq int(1 - \lambda) \leq cl[int(1 - \delta)] = 1 - \delta$.

Theorem 4.09: Let $\{A_\alpha\}_{\alpha\in\wedge}$ be a family of subsets of an fts (X,T) such that each $\mu_{A_\alpha}$is fuzzy $g^{\#\#}$-open . If $\wedge_{\alpha\in\wedge}\{A_\alpha\} \neq \emptyset$ and each $A_\alpha$is a fuzzy $g^{\#\#}$- connected subset of (X,T). Then $\vee_{\alpha\in\wedge}\{A_\alpha\}$ is also a fuzzy super $g^{\#\#}$- connected subset of (X,T).
Proof: Proof omitted

Theorem 4.10: If A and B are fuzzy super $g^{\#\#}$-connected subsets of an fts (X, T) and $\frac{int[\mu_B]}{A} \neq 0$ or $\frac{int[\mu_A]}{A} \neq 0$, $A \vee B$ is a fuzzy super $g^{\#\#}$-connected subsets of (X,T).
Proof: Proof omitted

Theorem 4.11: If $\{A_\alpha\}_{\alpha\in\wedge}$ be a family of fuzzy super $g^{\#\#}$-connected subset of an fts (X,T) such that $int[\wedge_{\alpha\in\wedge}\{A_\alpha\}] \neq 0$, then $\vee_{\alpha\in\wedge}\{A_\alpha\}$ is also a fuzzy super $g^{\#\#}$- connected subset of (X,T).
Proof: Proof omitted

Theorem 4.12: Let ft (X,T) is fuzzy super $g^{\#\#}$-connected and Let C is a fuzzy super $g^{\#\#}$-connected subset of (X,T). Let $X - C$ contain a set W such that $\frac{\mu_w}{X-C}$ is a fuzzy $g^{\#\#}$-open set in the fuzzy subspace $X - C$ of (X,T). Then $C \vee W$ is a fuzzy super $g^{\#\#}$-connected subset of (X,T).
Proof: Proof omitted

Theorem 4.13: If A and B are subsets of (X,T) and $\mu_A \leq \mu_B \leq cl[\mu_A]$. A is a fuzzy super $g^{\#\#}$-connected subset of (X,T), then B is also fuzzy super $g^{\#\#}$-connected subset of (X,T)
Proof: If assume B is not a fuzzy super $g^{\#\#}$-connected subset of (X,T), then there exist fuzzy $g^{\#\#}$-open sets $\frac{\delta_1}{B} \neq 0$, $\frac{\delta_2}{B} \neq 0$ and $\frac{\delta_1}{B} + \frac{\delta_2}{B} = 1$. Let us prove that $\frac{\delta_1}{A} \neq 0$. If $\frac{\delta_1}{A} = 0$, then $\delta_1 + \mu_A \leq 1$. It implies that $\delta_1 + cl(\mu_A) \leq 1$, since $\mu_B \leq cl(\mu_A)$, therefore $\delta_1 + \mu_B \leq 1$. This implies that $\frac{\delta_1}{B} = 0$, which contradicts the fact that $\frac{\delta_1}{B} \neq 0$, therefore $\frac{\delta_1}{A} \neq 0$. Similarly let us prove that $\frac{\delta_2}{A} \neq 0$. If $\frac{\delta_2}{A} = 0$, then $\delta_2 + \mu_A \leq 1$. Now $\frac{\delta_1}{B} \neq 0$, $\frac{\delta_2}{B} \neq 0$, $\frac{\delta_1}{B} + \frac{\delta_2}{B} = 1$ and $\mu_A \leq \mu_B$ implies that $\frac{\delta_1}{A} + \frac{\delta_2}{A} = 1$. Therefore A is not super $g^{\#\#}$-connected, which contradicts the fact.

Theorem 4.14: Let (X,T) and (Y,σ)be any two fuzzy super $g^{\#\#}$-connected spaces which are product related. Then $(X * Y, T * \sigma)$ is a fuzzy super $g^{\#\#}$-connected space.
Proof: Proof omitted

5. FUZZY STRONGLY $g^{\#\#}$-CONNECTEDNESS IN FTS

Definition 5.01: A fts (X,T) is said to be Fuzzy strongly $g^{\#\#}$-connected, if it has no non zero fuzzy $g^{\#\#}$-closed sets $\delta_1$and δ_2 such that $\delta_1 + \delta_2 \leq 1$. If (X,T) is not fuzzy strongly $g^{\#\#}$-connected, then it is said to be weakly $g^{\#\#}$-connected.

Theorem 5.02: Let fts (X,T) be fuzzy strongly $g^{\#\#}$-connected, iff it has non zero fuzzy $g^{\#\#}$-open sets, like $\alpha$and β such that $\alpha + \beta \geq 1$.

Proof: An fts (X,T) is fuzzy $g^{\#\#}$- weakly disconnected. If it has non zero fuzzy $g^{\#\#}$- closed sets $\delta_1$and δ_2 such that $\delta_1 + \delta_2 \leq 1$. If it has non zero fuzzy $g^{\#\#}$-open sets $= \delta'_1$, $\beta = \delta'_2$ such that $\alpha \neq 1, \beta \neq 1$ and $\alpha + \beta \geq 1$. By hypothesis, the fuzzy strongly $g^{\#\#}$-connected is fuzzy $g^{\#\#}$-connected, but the converse is not true. Therefore the fuzzy strongly $g^{\#\#}$-connected and fuzzy super $g^{\#\#}$-connected are unrelated.
Example 5.03: If $X = [0,1]$ and for $0 \leq x \leq 1$, $\alpha(x) = 1/3$ and $T_1(X) = \{0,1,\alpha\}$ and $T_2(X) = \{0,1,\alpha'\}$, then $(X, T_1(X))$ is fuzzy $g^{\#\#}$-connected and fuzzy super $g^{\#\#}$-connected, but not fuzzy strongly $g^{\#\#}$-connected. And $(X, T_2(X))$ is fuzzy strongly $g^{\#\#}$-connected, but not fuzzy super $g^{\#\#}$-connected.

Theorem 5.04: If $A < X$ and fts (X,T) is a fuzzy strongly $g^{\#\#}$-connected subset of X, iff for any fuzzy $g^{\#\#}$-open sets α and β in (X,T). $\mu_A \leq \alpha + \beta$ implies either $\mu_A \leq \alpha$ or $\mu_A \leq \beta$.
Proof: If A is not a fuzzy strongly $g^{\#\#}$-connected subset of (X,T), then there exist fuzzy $g^{\#\#}$-closed sets $\delta_1$and δ_2 in (X,T), such that (a) $\frac{\delta_1}{A} \neq 0$ (b) $\frac{\delta_2}{A} \neq 0$ and (c) $\frac{\delta_1}{A} + \frac{\delta_2}{A} \leq 1$. If we put $\alpha = 1 - \delta_1$ and $\beta = 1 - \delta_2$ then $\frac{\alpha}{A} = 1 - \frac{\delta_1}{A}$, $\frac{\beta}{A} = 1 - \frac{\delta_2}{A}$. Therefore $\mu_A \leq \alpha + \beta$ but $\mu_A \nleq \alpha$ and $\mu_A \nleq \beta$. Conversely if there exist an fuzzy $g^{\#\#}$-open sets $\alpha$and β such that $\mu_A \leq \alpha + \beta$, but $\mu_A \nleq \alpha$ and $\mu_A \nleq \beta$.Then $\frac{\alpha}{A} \neq 1$, $\frac{\beta}{A} \neq 1$ and $\frac{\alpha}{A} + \frac{\beta}{A} \geq 1$. Therefore A is not a fuzzy strongly $g^{\#\#}$-connected.

Theorem 5.05: If H is a subset of an fts (X,T), such that μ_H is a fuzzy $g^{\#\#}$-closed in (X,T), then (X,T) is fuzzy strongly $g^{\#\#}$-connected , which implies that H is a is a fuzzy strongly $g^{\#\#}$-connected subset of X.
Proof: Assume H is not a fuzzy strongly $g^{\#\#}$-connected subset of (X,T). Then there exist fuzzy $g^{\#\#}$-closed sets $\delta_1$and δ_2 in (X,T), such that (a) $\frac{\delta_1}{H} \neq 0$ (b) $\frac{\delta_2}{H} \neq 0$ and (c) $\frac{\delta_1}{H} + \frac{\delta_2}{H} \leq$, here (c) implies $(\delta_1 \wedge \mu_H) + (\delta_2 \wedge \mu_H) \leq 1$, where $\frac{\delta_1}{H} \neq 0$ and $\frac{\delta_2}{H} \neq 0$. So $\delta_1 \wedge \mu_H \neq 0$ and $\delta_2 \wedge \mu_H \neq 0$. Therefore (X,T) is not fuzzy strongly $g^{\#\#}$-connected and which contradicts the fact.

Theorem 5.06: Let (X,T) be fts and the function $f: X \to Y$ is continuous, then (X,T) is fuzzy strongly $g^{\#\#}$-connected. Which implies that Y is fuzzy strongly $g^{\#\#}$-connected.
Proof: By the definition of $g^{\#\#}$- continuous mapping , A function $f: X \to Y$ is $fg^{\#\#}$-continuous iff the inverse image of every closed fuzzy set in Y is $g^{\#\#}$-closed fuzzy set in X.Also by the definition of fuzzy strongly $g^{\#\#}$-continuous function , A function $f: X \to Y$ is fuzzy strongly $g^{\#\#}$-continuous, iff the inverse image of every $g^{\#\#}$-closed fuzzy set in Y is $g^{\#\#}$-closed fuzzy set in X. Therefore f is fuzzy strongly $g^{\#\#}$-connected in (Y,σ) and also in (X,T).

Theorem 5.07: A finite product of fuzzy strongly $g^{\#\#}$- connected space is fuzzy strongly $g^{\#\#}$- connected.
Proof: Proof omitted

Observation 5.08: An finite product of fuzzy strongly $g^{\#\#}$- connected space need not be fuzzy strongly $g^{\#\#}$-connected.

Example 5.09: Let $X_n = [0,1]$, where $n = 1,2,3 \ldots$. Let $\alpha_n(x) = \frac{1}{2}\left\{\frac{n}{(n+1)}\right\}$, if $x \in X$. And $T(X_n) = \{0,1,\alpha_n\}$, where $n = 1,2,3 \ldots$. Then each (X,T) is . But the fuzzy $g^{\#\#}$- connected space $X = \wedge_{n=1}^{\infty} X_n$ is not as $T(X)$ contains a member $\vee_{n=1}^{\infty} P_n^{-1}(\alpha_n) \neq 1$, such that $\left\{\vee_{n=1}^{\infty} P_n^{-1}(\alpha_n)\right\}(x) = \frac{1}{2}$, where $x \in X$.

6. CONCLUSION

In this paper, we studied the new results on the spaces and investigated the extension of connected spaces like, $g^{\#\#}$-connected, $g^{\#\#}$-super connected, $g^{\#\#}$- strongly connected. We observed few results like, the product of $fg^{\#\#}$-connected spaces need not be $fg^{\#\#}$-connected spaces. Fuzzy $g^{\#\#}$-connected and $fg^{\#\#}$-super connectedness ate not related to each other. An infinite product of $fg^{\#\#}$-strongly connected spaces need not be $fg^{\#\#}$- strongly connected, where as it is possible in general topology. This work further lead to the study of disconnectedness of fuzzy $g^{\#\#}$-closed sets.

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