ON FUZZY SUPRA PRE $\sigma$-BAIRE SPACES
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ABSTRACT: In this paper, the concepts of fuzzy supra pre $\sigma$-nowhere dense set, fuzzy supra pre $\sigma$-first category set and fuzzy supra pre $\sigma$-second category set in fuzzy topological spaces are introduced and studied. By means of fuzzy supra pre $\sigma$-nowhere dense sets, the concept of fuzzy supra pre $\sigma$-Baire space is defined and several characterizations of fuzzy supra pre $\sigma$-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy supra pre dense set, fuzzy supra pre nowhere dense set, fuzzy supra pre $F_\sigma$-set, fuzzy supra pre $G_\delta$-set, fuzzy supra pre $\sigma$-nowhere dense set, fuzzy supra pre $\sigma$-first category, fuzzy supra pre $\sigma$-second category, fuzzy supra pre $\sigma$-Baire space.

1. INTRODUCTION
Dealing with the uncertainties in our real life phenomena has been a critical matter in recent years. Classical set theory was not suitable in such cases. Since 1965, Zadeh [1] introduced a more suitable set theory providing the concept "fuzzy set". A large community of Mathematics has put their relentless efforts to the investigation of various concepts of general topology in fuzzy setting likely, [2-5]. In 2012, Selvi and Dhari, [6] introduced the concept of pre* open (supra – preopen) sets in general topology. In 2016, Hakeem A.Othman [7] introduced the concept of fuzzy supra open sets in supra topology. The concept of fuzzy supra $\sigma$-Baire spaces were introduced and studied by Poongothai and Thangaraj [8]. In this paper, the concepts of fuzzy supra pre $\sigma$-nowhere dense sets and fuzzy supra pre $\sigma$-Baire spaces are introduced and studied. Several characterizations of fuzzy supra pre $\sigma$-Baire spaces are studied.

2. PRELIMINARIES
Definition 2.1 [8]: A fuzzy topology $T$ on a set $X$ is a family of fuzzy sets in $X$ such that
0$_X$, 1$_X$ $\in$ $T$;
$A, B \in T \Rightarrow A \wedge B \in T$ and
$A_i \in T \Rightarrow \forall A_i \in T$

The pair $(X, T)$ is called a fuzzy topological space (FTS). The elements of $T$ are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.2 [4]: The closure and the interior of a fuzzy set $A$ in FTS $(X, T)$ are denoted and defined respectively as
$Cl (A) = \wedge \{B: A \leq B, B \text{ is a fuzzy closed set in } X\}$,
$Int (A) = \vee \{B: B \leq A, B \text{ is a fuzzy open set in } X\}$.

Definition 2.3[1]: A collection $T^*$ of fuzzy open sets in a set $X$ is called a fuzzy supra topology on $X$ if the following conditions are satisfied:
(1) $0_X, 1_X \in T$ and (2) $A_i \in T \Rightarrow \forall A_i \in T$

The pair $(X, T^*)$ is called a fuzzy supra topological space (FSTS). The elements of $T^*$ are called fuzzy supra open sets (FSOS) and the complement of a fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp.fuzzy supra closed sets) of the FSTS $(X, T^*)$ is denoted by FSOS($X$) (resp.FSCS($X$)).

Remark 2.4 (4): Every FTS is a FSTS.

If $(X, T^*)$ is an associated FSTS with the FTS $(X, T)$ (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS $(X, T)$ is fuzzy supra open (closed) set in the FSTS $(X, T^*)$.

Definition 2.5 [7]: Let $(X, T^*)$ is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as $Cl^*(A) = \wedge \{B: A \leq B, B \text{ is a fuzzy supra closed set in } X\}$,
$Int^*(A) = \vee \{B: B \leq A, B \text{ is a fuzzy supra open set in } X\}$.

Remark 2.6 [7]: The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A.
The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set containing A.
If $(X, T^*)$ is an associated FSTS with the FTS $(X, T)$ and A is any fuzzy set in X, then
$Int (A) \leq Int^*(A) \leq A \leq Cl^*(A) \leq Cl (A)$.

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in
The following theorem.

**Theorem 2.7** [4]: For any fuzzy sets $A$ and $B$ in a FSTS $(X, T^*)$, $A \in FSCS(X) \Rightarrow Cl^*(A) = A, \ A \in FSOS(X) \Rightarrow Int^*(A) = A$.

$A \leq B, \Rightarrow Cl^*(A) \leq Cl^*(B)$ and $Int^*(A) \leq Int^*(B)$;

$Cl^*(Cl^*(A)) = Cl^*(A)$ and $Int^*(Int^*(A)) = Int^*(A)$;

$Cl^*(A \lor B) \geq Cl^*(A) \lor Cl^*(B)$;

$Cl^*(A \land B) \leq Cl^*(A) \land Cl^*(B)$;

$Int^*(A \lor B) \geq Int^*(A) \lor Int^*(B)$;

$Int^*(A \land B) \leq Int^*(A) \land Int^*(B)$;

$Cl^*(A^C) = (Int^*(A))^C, Int^*(A^C) = (Cl^*(A))^C$.

**Definition 2.8** [5]: A fuzzy set $\lambda$ of fuzzy space $X$ is called fuzzy supra preopen (supra preclosed) set if $\lambda \leq int^* cl^* \lambda (cl^* int^* \lambda \leq \lambda)$. The class of all fuzzy supra-preopen (supra - preclosed) sets in $X$ will as denoted be $SFPO(X)$ ($SFPC(X)$).

**Definition 2.9** [5]: The closure and the interior of a fuzzy set $A$ in FTS $(X, T)$ are denoted and defined respectively as

$sp Cl(A) = \land \{B : B \leq A, B \text{ is a fuzzy supra preclosed set of } X\}$,

$sp Int(A) = \lor \{B : B \leq A, B \text{ is a fuzzy supra open set of } X\}$.

**Definition 2.10** [5]: Let $A$ be a fuzzy supra set of a fuzzy supra topological space $(X, T)$. Then (i) $1 - pcl^*(A) = pint^*(1 - A)$ (ii) $1 - pint^*(A) = pcl^*(1 - A)$.

**Definition 2.11** [9]: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called fuzzy supra $F_\sigma$ set in $(X, T^*)$ if $A = \lor_{i=1}^{\infty} A_i$, where $1 - A_i \in T^*$ for $i \in I$.

**Definition 2.12** [9]: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called a fuzzy supra $G_\delta$ set in $(X, T^*)$ if $A = \land_{i=1}^{\infty} A_i$, where $A_i \in T^*$ for $i \in I$.

**Definition 2.13** [9]: Let $(X, T^*)$ be a fuzzy supra topological space. A fuzzy set $A$ in $(X, T^*)$ is called a fuzzy supra $\sigma$ - nowhere dense set if $A$ is a fuzzy supra $F_\sigma$ set in $(X, T^*)$ such that $int^*(A) = 0$.

**Definition 2.14** [9]: Let $(X, T^*)$ be a fuzzy supra topological space. A fuzzy set $A$ in $(X, T^*)$ is called fuzzy supra $\sigma$ - first category if $A = \lor_{i=1}^{\infty} (A_i)$, where $(A_i)$'s are fuzzy supra $\sigma$ - nowhere dense sets in $(X, T^*)$. Any other fuzzy set in $(X, T^*)$ is said to be fuzzy supra $\sigma$ - second category in $(X, T^*)$.

3. **FUZZY SUPRA PRE $\sigma$-NOWHERE DENSE SETS**

**Definition 3.1**: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called fuzzy supra pre dense if there exists no fuzzy supra pre closed set $B$ in $(X, T^*)$ such that $A < B < 1$. That is, $pCl^*(A) = 1$.

**Definition 3.2**: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called fuzzy supra pre $F_\sigma$ set in $(X, T^*)$ if $A = \lor_{i=1}^{\infty} A_i$, where $(A_i)$'s are fuzzy supra pre-closed sets in $(X, T^*)$.

**Definition 3.3**: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called a fuzzy supra pre $G_\delta$ set in $(X, T^*)$ if $A = \land_{i=1}^{\infty} A_i$, where $(A_i)$'s are fuzzy supra pre-open sets in $(X, T^*)$.

**Definition 3.4**: A fuzzy set $A$ in a FSTS $(X, T^*)$ is called fuzzy supra pre nowhere dense if there exists no fuzzy supra pre-open set $B$ in $(X, T^*)$ such that $B < pcl^*(A)$. That is, $pint^* pcl^*(A) = 0$.

**Definition 3.5**: Let $(X, T^*)$ be a fuzzy supra topological space. A fuzzy set $A$ in $(X, T^*)$ is called a fuzzy supra $\sigma$ - nowhere dense set if $A$ is a fuzzy supra pre $F_\sigma$ set in $(X, T^*)$ such that $pint^*(A) = 0$.

**Example 3.6**: Let $X = \{a, b\}$ be a set with a fuzzy supra topology $(X, T^*)$. Then the fuzzy sets $A = \{a_7, b_7\}, B = \{a_7, b_6\}, C = \{a_6, b_8\}$. Then...
Now, consider the fuzzy set
\[
\alpha = (1 - A \lor B) \lor (1 - B \lor C) \lor (1 - B \land C)
\]
in \((X, T^*)\). Then \(\alpha\) is fuzzy supra pre \(F_\sigma\) - set in \((X, T^*)\) and \(\text{pint}^*(\alpha) = 0\) and hence \(\alpha\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Remarks 3.7:** If \(A\) and \(B\) are fuzzy supra pre \(\sigma\) - nowhere dense sets in a fuzzy supra topological space \((X, T^*)\), then \(A \lor B\) need not be a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\). For, consider the following example:

**Example 3.8:** Let \(X = \{a, b, c\}\) be a set with a fuzzy supra topology \((X, T^*)\). Then the sets \(A = \{a, b, c\}\), \(B = \{a\}\), \(C = \{a, b, c\}\).

\[ T^* = \{0, A, B, C, A \lor B, B \lor C, B \land C, 1 - A, 1 - B, 1 - C, 1 - A \lor B, 1 - B \lor C, 1 - B \land C, 1\} \]

Now, consider the fuzzy set \(\alpha = [(1 - B) \lor (1 - (A \lor B))] \lor [1 - B \lor C]\) and \(\beta = [(1 - A) \lor (1 - C)]\) are fuzzy supra pre \(F_\sigma\) - sets in \((X, T^*)\). Also \(\text{pint}^*(\alpha) = 0\) and \(\text{pint}^*(\beta) = 0\). Therefore \(\alpha\) and \(\beta\) are fuzzy supra pre \(\sigma\) - nowhere dense sets in \((X, T^*)\). Clearly \((\alpha \lor \beta)\) is a fuzzy supra pre \(F_\sigma\) - set in \((X, T^*)\). On computation, \(\text{pint}^*(\alpha \lor \beta) = (B \land C) \neq 0\), implies that \((\alpha \lor \beta)\) is not a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Proposition 3.9:** In a fuzzy supra topological space \((X, T^*)\) a fuzzy set \(A\) is fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\) if and only if \(1 - A\) is a fuzzy supra pre dense and fuzzy supra pre \(G_\delta\) - set in \((X, T^*)\).

**Proof:** Let \(A\) be a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\). Then \(A = \bigvee_{i=1}^{\infty} A_i\), where \(1 - A_i \in T^*\) for \(i \in I\) and \(\text{pint}^*(A) = 0\). Then 
\[ 1 - \text{pint}^*(A) = 1 \]
implies that 
\[ \text{pcl}^*(1 - A) = 1 \]
Also 
\[ (1 - A) = 1 - V_i^{\infty} (A_i) = \bigvee_{i=1}^{\infty} (1 - A_i) \]
where \((1 - A_i) \in T^*\), for \(i \in I\). Hence we have \((1 - A)\) is a fuzzy supra pre dense and fuzzy supra pre \(G_\delta\) - set in \((X, T^*)\).

Conversely, let \(A\) be a fuzzy supra pre dense and fuzzy supra pre \(G_\delta\) - set in \((X, T^*)\). Then \(A = \bigvee_{i=1}^{\infty} A_i\) where \(1 - A_i \in T^*\), for \(i \in I\). Hence 
\[ (1 - A) = 1 - \bigvee_{i=1}^{\infty} (1 - A_i) = 1 - \bigvee_{i=1}^{\infty} (A_i) \]
and \(\text{pint}^*(1 - A) = 1 - \text{pcl}^*(A) = 1 - 1 = 0\). Since \(A\) is a fuzzy supra pre dense, therefore \(1 - A\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Proposition 3.10:** If \(A\) is a fuzzy supra pre dense set in \((X, T^*)\) such that \(B \leq (1 - A)\), where \(B\) is a fuzzy supra pre \(F_\sigma\) - set in \((X, T^*)\), then \(B\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Proof:** If \(A\) is a fuzzy supra pre dense set in \((X, T^*)\) such that \(B \leq (1 - A)\). Now \(B \leq (1 - A)\) implies that 
\[ \text{pint}^*(B) \leq \text{pint}^*(1 - A) = 1 - \text{pcl}^*(A) = 1 - 1 = 0 \]
and hence \(\text{pint}^*(B) = 0\). Therefore \(B\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Proposition 3.11:** If \(A\) is a fuzzy supra pre \(F_\sigma\) - set and fuzzy supra pre nowhere dense set in \((X, T^*)\), then \(A\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Proof:** Now \(A \leq \text{pcl}^*(A)\) for any fuzzy set in \((X, T^*)\). Then, \(\text{pint}^*(A) \leq \text{pint}^*\text{pcl}^*(A)\). Since \(A\) is a fuzzy supra pre nowhere dense set in \((X, T^*)\), 
\[ \text{pint}^*\text{pcl}^*(A) = 0 \]
and hence \(\text{pint}^*(A) = 0\) and \(A\) is a fuzzy supra pre \(F_\sigma\) - set implies that \(A\) is a fuzzy supra pre \(\sigma\) - nowhere dense set in \((X, T^*)\).

**Definition 3.12:** Let \((X, T^*)\) be a fuzzy supra topological space. A fuzzy set \(A\) in \((X, T^*)\) is called fuzzy supra pre \(\sigma\) - first category if \(A = \bigvee_{i=1}^{\infty} (A_i)\) where \(A_i\)'s are fuzzy supra pre \(\sigma\) - nowhere dense sets in \((X, T^*)\). Any other fuzzy set in \((X, T^*)\) is said to be fuzzy supra pre \(\sigma\) - second category in \((X, T^*)\).
Definition 3.13: Let $A$ be a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$. Then $1 - A$ is called a fuzzy supra pre $\sigma$—residual set in $(X, T^*)$.

Definition 3.14: A fuzzy supra topological space $(X, T^*)$ is called fuzzy supra pre $\sigma$—first category if the fuzzy set $1_X$ is a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$. That is, $1_X = \text{V}^\infty_{i=1} (A_i)$ where $(A_i)$s are fuzzy supra pre $\sigma$—nowhere dense sets in $(X, T^*)$. Otherwise, $(X, T^*)$ will be called a fuzzy supra pre $\sigma$—second category space.

Proposition 3.15: If $A$ is a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$, then there is a fuzzy supra pre $F_\sigma$-set $B$ in $(X, T)$ such that $A \subseteq B$.

Proof: Let $A$ be a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$. Then $A = \text{V}^\infty_{i=1} (A_i)$, where $(A_i)$s are fuzzy supra pre $\sigma$—nowhere dense sets in $(X, T^*)$. Now, $\text{V}^\infty_{i=1} (A_i)\{ (i=1 \text{ to } \infty) \}$ are fuzzy supra pre open sets in $(X, T^*)$. Then $C = \text{A}^\infty_{i=1} (1 - p \text{ cl}^* (A_i))$ is a fuzzy supra pre $G_\sigma$-set in $(X, T^*)$ and $1 - C = 1 - [\text{A}^\infty_{i=1} (1 - p \text{ cl}^* (A_i))] = [\text{V}^\infty_{i=1} p \text{ cl}^*(A_i)]$.

Now $\text{A}^\infty_{i=1} (A_i) \leq p \text{ cl}^* (A_i)$, implies that $\text{V}^\infty_{i=1} (A_i) \leq [\text{V}^\infty_{i=1} p \text{ cl}^* (A_i)]$. Hence $A = \text{V}^\infty_{i=1} (A_i) \leq [\text{V}^\infty_{i=1} p \text{ cl}^* (A_i)] = [1 - B]$. That is, $A \leq [1 - B]$ and $[1 - B]$ is a fuzzy supra pre $F_\sigma$-set in $(X, T^*)$. Let $B = [1 - C]$. Hence, if $A$ is a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$, then there is a fuzzy supra pre $F_\sigma$-set $B$ in $(X, T)$ such that $A \leq B$.

Proposition 3.16: If $A$ is a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$, then there is a fuzzy supra pre $F_\sigma$-set $B$ in $(X, T^*)$ such that $A \leq B \leq p \text{ cl}^* (A)$, where $B$ is a fuzzy supra pre $F_\sigma$-set in $(X, T^*)$.

Proof: Let $A$ be a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$. Then $A = \text{V}^\infty_{i=1} (A_i)$, where $(A_i)$s are fuzzy supra pre $\sigma$—nowhere dense sets in $(X, T^*)$. Now $\text{V}^\infty_{i=1} (A_i)\{ (i=1 \text{ to } \infty) \}$ are fuzzy supra pre open sets in $(X, T^*)$. Then $B = \text{A}^\infty_{i=1} (1 - p \text{ cl}^* (A_i))$ is a fuzzy supra pre $G_\sigma$-set in $(X, T^*)$ and $1 - B = 1 - [\text{A}^\infty_{i=1} (1 - p \text{ cl}^* (A_i))] = [\text{V}^\infty_{i=1} p \text{ cl}^*(A_i)]$.

Now $A = \text{V}^\infty_{i=1} (A_i) \leq [\text{V}^\infty_{i=1} p \text{ cl}^* (A_i)] = [1 - B]$. That is, $A \leq [1 - C]$ and $[1 - C]$ is a fuzzy supra pre $F_\sigma$-set in $(X, T^*)$. Let $B = [1 - C]$. Hence, if $A$ is a fuzzy supra pre $\sigma$—first category set in $(X, T^*)$, then there is a fuzzy supra pre $F_\sigma$-set $B$ in $(X, T^*)$ such that $A \leq B \leq p \text{ cl}^* (A)$, where $B$ is a fuzzy supra pre $F_\sigma$-set in $(X, T^*)$.

Proposition 3.17: If $A$ is a fuzzy supra pre closed set in a fuzzy supra topological space $(X, T^*)$ and if $\text{pint}^* (A) = 0$, then $A$ is a fuzzy supra pre $\sigma$—nowhere dense set in $(X, T^*)$.

Proof: Let $A$ be a fuzzy supra pre closed set in $(X, T^*)$. Then we have $p \text{ cl}^* (A) = A$. Now $\text{pint}^* [p \text{ cl}^* (A)] = \text{pint}^* (A)$ and $\text{pint}^* (A) = 0$, implies that $A$ is a fuzzy supra pre $\sigma$—nowhere dense set in $(X, T^*)$.

4. Fuzzy Supra Pre $\sigma$-BAIRE SPACE

Definition 4.1: Let $(X, T^*)$ be a fuzzy supra topological space. Then $(X, T^*)$ is called a fuzzy supra pre $\sigma$—Baire Space if $\text{pint}^* (\text{V}^\infty_{i=1} (A_i))$ where $(A_i)$s are fuzzy supra pre $\sigma$—nowhere dense sets in $(X, T^*)$.

Example 4.2: Let $X = \{a, b, c\}$ be a set with a fuzzy supra topology $(X, T^*)$. Then the fuzzy sets $A = \{a, b, c\}$, $B = \{a, b, c\}$, $c = \{a, b, c\}$

Then $T^* = \{0, A, B, C, A \lor B, A \lor C, B \lor C, B \land C, 1 - A, 1 - B, 1 - C, 1 - A \lor B, 1 - B \lor C, 1 - B \land C, 1\}$. Now, consider the fuzzy set $\alpha = (1 - A \lor B) \lor (1 - B \lor C) \lor (1 - B \land C)$ in $(X, T^*)$. Then $\alpha$ is fuzzy supra pre $F_\sigma$—set in...
and \( \text{pint}^*(\alpha \vee \beta) = 0 \) and hence \( \beta \) is a fuzzy supra pre \( \sigma \) - nowhere dense set in \((X, T^*)\). Then \( \alpha \) and \( \beta \) are fuzzy supra pre \( \sigma \) - nowhere dense set in \((X, T^*)\) and also \( \text{pint}^*(\alpha \vee \beta) = 0 \) and therefore \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space.

**Proposition 4.3:** Let \((X, T^*)\) be a fuzzy supra topological space. Then the following are equivalent:

\( (X, T^*) \) is a fuzzy supra pre \( \sigma \) - Baire Space.

1. \( \text{pint}^*(A) = 0 \) for every fuzzy supra pre \( \sigma \) - first category set \( A \) in \((X, T^*)\).

2. \( \text{pcl}^*(B) = 1 \) for every fuzzy supra pre \( \sigma \) - residual set \( B \) in \((X, T^*)\).

**Proof:** (1) \( \Rightarrow \) (2). Let \( A \) be a fuzzy supra pre \( \sigma \) - first category set \( A \) in \((X, T^*)\). Then \( A = \left( \text{V}_{i=1}^{\infty} (A_i) \right) \) where \( (A_i) \)'s are fuzzy supra pre \( \sigma \) - nowhere dense sets in \((X, T^*)\). Then, we have \( \text{pint}^*(A) = \text{pint}^*(\text{V}_{i=1}^{\infty} (A_i)) = 0 \).

Since \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space, \( \text{pint}^*(\text{V}_{i=1}^{\infty} (A_i)) = 0 \). Hence \( \text{pint}^*(A) = 0 \) for any fuzzy supra pre \( \sigma \) - first category set \( A \) in \((X, T^*)\).

(2) \( \Rightarrow \) (3). Let \( B \) be a fuzzy supra pre \( \sigma \) - residual set \( B \) in \((X, T^*)\). Then \( (1 - B) \) is a fuzzy supra pre \( \sigma \) - first category set \( B \) in \((X, T^*)\). By hypothesis, \( \text{pint}^*(1 - B) = 0 \). Then \( 1 - \text{pcl}^*(B) = 0 \). Hence \( \text{pcl}^*(B) = 1 \) for any fuzzy supra pre \( \sigma \) - residual set \( B \) in \((X, T^*)\).

(3) \( \Rightarrow \) (1). Let \( A \) be a fuzzy supra pre \( \sigma \) - first category set \( A \) in \((X, T^*)\). Then \( A = \left( \text{V}_{i=1}^{\infty} (A_i) \right) \) where \( (A_i) \)'s are fuzzy supra pre \( \sigma \) - nowhere dense sets in \((X, T^*)\). Now \( A \) is a fuzzy supra pre \( \sigma \) - first category set in \((X, T^*)\) implies that \( (1 - A) \) is a fuzzy supra pre \( \sigma \) - residual set in \((X, T^*)\). By hypothesis, we have \( \text{pcl}^*(1 - A) = 1 \). Then \( 1 - \text{pint}^*(A) = 1 \). Hence \( \text{pint}^*(A) = 0 \). That is, \( \text{pint}^*(\text{V}_{i=1}^{\infty} (A_i)) = 0 \) where \( (A_i) \)'s are fuzzy supra pre \( \sigma \) - nowhere dense sets in \((X, T^*)\). Hence \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space.

**Proposition 4.4:** If \( \text{pcl}^*(\text{A}_{i=1}^{\infty} (A_i)) = 1 \), where \( (A_i) \)'s are fuzzy supra pre dense and fuzzy supra pre \( \sigma \) - Baire Space.

**Proof:** Now \( \text{pcl}^*(\text{A}_{i=1}^{\infty} (A_i)) = 1 \), implies that \( 1 - \text{pcl}^*(\text{A}_{i=1}^{\infty} (A_i)) = 0 \), which implies that \( \text{pint}^*(\text{V}_{i=1}^{\infty} (1 - A_i)) = 0 \). Let \( B_1 = 1 - A_i \). Then \( \text{pint}^*(\text{V}_{i=1}^{\infty} (B_i)) = 0 \). Since \( A_i \) is a fuzzy supra pre dense and fuzzy supra pre \( \sigma \) - Baire Space. Then \( \text{pint}^*(\text{V}_{i=1}^{\infty} (B_i)) = 0 \), which implies that \( 0 = 1, \) a contradiction. Hence \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space.

**Proposition 4.5:** If the fuzzy supra topological space \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space, then \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - second category space.

**Proof:** Let \((X, T^*)\) be a fuzzy supra pre \( \sigma \) - Baire Space. Then \( \text{pint}^*(\text{V}_{i=1}^{\infty} (A_i)) = 0 \) where \( (A_i) \)'s are fuzzy supra pre \( \sigma \) - Baire Space. Then \( \text{V}_{i=1}^{\infty} (A_i) = \text{I}_X \) implies that \( \text{pint}^*(\text{V}_{i=1}^{\infty} (A_i)) = \text{int}^*1_X = 1_X \), which implies that \( 0 = 1, \) a contradiction. Hence \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - second category space.

**Proposition 4.6:** If the fuzzy supra topological space \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - Baire Space, then \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - second category space.

**Proof:** Let \((X, T^*)\) be a fuzzy supra pre \( \sigma \) - Baire Space. Then \( (1 - A_i) \) is a fuzzy supra pre \( \sigma \) - Baire Space. Then \( \text{pint}^*(\text{V}_{i=1}^{\infty} (1 - A_i)) = 0 \). Therefore \((X, T^*)\) is a fuzzy supra pre \( \sigma \) - second category space. Hence
where $1 - A_i$'s are fuzzy supra pre nowhere dense set in $(X, T^*)$. Hence $(X, T^*)$ is not a fuzzy supra pre first category space. Therefore $(X, T^*)$ is a fuzzy supra pre second category space.

**Proposition 4.7:** If the fuzzy supra topological space $(X, T^*)$ is a fuzzy supra pre first category space, then $(X, T^*)$ is not a fuzzy supra pre Baire Space.

**Proof:** Let the fuzzy supra topological space $(X, T^*)$ is a fuzzy supra pre first category space. Then $V_{i=1}^\infty (A_i) = 1_x$, where $(A_i)$'s are fuzzy supra pre nowhere dense sets in $(X, T^*)$. Now $\pi^\ast(V_{i=1}^\infty (A_i)) = \pi^\ast(1_x) = 1 \neq 0$. Hence by definition, $(X, T^*)$ is not a fuzzy supra pre Baire Space.

**Proposition 4.8:** If the fuzzy supra topological space $(X, T^*)$ is a fuzzy supra pre Baire Space and if $V_{i=1}^\infty (A_i) = 1$, then there exists at least one fuzzy supra pre $F_\sigma$ set $A_i$ such that $\pi^\ast(A_i) \neq 0$.

**Proof:** Suppose that $\pi^\ast(A_i) = 0$, for $i = 1$ to $\infty$, where $(A_i)$'s are fuzzy supra pre nowhere dense sets in $(X, T^*)$. Then $V_{i=1}^\infty (A_i) = 1$. Implies that $\pi^\ast(V_{i=1}^\infty (A_i)) = \pi^\ast(1) = 1 \neq 0$, a contradiction to $(X, T^*)$ being a fuzzy supra pre Baire Space. Hence $\pi^\ast(A_i) \neq 0$, for at least one fuzzy supra pre $F_\sigma$ set $A_i$ in $(X, T^*)$.

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