Tight Focusing Properties of Phase Modulated Longitudinally Polarized Doughnut Gaussian Beam

K.Prabakaran^{*1}, K.B.Rajesh², S.Ramachandran³, M.Jayaprakash³, V.Aroulmoji^{4,5}

¹Department of Physics, Mahendra Arts and Science College (Autonomous), Namakkal, Tamilnadu, India ²Department of Physics, Chikkanna Government Arts College, Trippur, Tamilnadu, India ³Department of Physics, Arignar Anna College (Arts & Science), Krishnagiri, Tamilnadu, India ⁴Center for Research and Development, Mahendra Educational Institutions, Namakkal, Tamil Nadu, India ⁵Mahendra Engineering College, Mahendhirapuri, Mallasamudram - 637 503, India

ABSTRACT: We investigated the focusing properties of a phase modulated radially polarized doughnut Gaussian beam by a high numerical aperture (NA) lens based on vector diffraction theory. We observed that our proposed system generates a sub wavelength focal spot of 0.42 λ having large uniform focal depth of 5 λ . This kind of non diffracting focal segments is called as dark channel, which may have application in atom trapping, nanolithography, trapping and manipulating particles.

KEYWORDS: Radially polarized beam, Vector Diffraction theory, Doughnut gaussian beam, Optical Trapping.

DOI: 10.29294/IJASE.4.3.2018.651-655

© 2018 mahendrapublications.com, All rights reserved

1. INTRODUCTION

Recently, research on longitudinally polarized laser beams has become a very hot topic due to its potential applications in various areas, such as particle acceleration [1], second harmonic generation [2], fluorescent imaging [3], optical data storage [4] and optical trapping [5]. A radially polarized beam can be focused to generate a strong longitudinal electric field at the focal plane, resulting in a focal spot sharper than that for a linearly polarized beam [6,7]. In theory, a radially polarized Bessel-Gauss beam is focused with a binary optical element and a high numerical aperture (NA) objective to produce a sub-wavelength longitudinally polarized light needle with a depth of focus (DOF) about 4λ [8]. Subsequently, several approaches, such as a narrow width annular beam [9], dual-beam focusing [10] and higherorder beams [11-13], have been employed to realize smaller focal spot sizes and longer DOF. It is of great importance to design the diffractive optical element (DOE) efficiently and accurately. Much work has been done to realize the desired goal methods of designing DOE [14-22,23]. Huang et al [24] formulated a method that combines the global-search-optimization (GSO) algorithm and the tight focusing properties of radially polarized light to design a DOE.

However, to the best of our knowledge, most of the previous research has dealt with beams with either the uniform [25,26] or Laguerre–Gaussian pupil apodization functions [25,27,28]. There has been hardly any investigation of incident beams with Gaussian and Bessel–Gaussian pupil apodization functions, much less a comparison of the optical shaping generated from these beams. Wang et.al [29] proposed the double-mode radially polarized (RP) beams can be generated conveniently. But the vector beams possess a uniform pupil apodization

function; therefore, the corresponding optical shaping formed by those vector beams belongs to the case of uniform apodization pattern [25]. In fact, not only can double-mode vector beams with uniform pupil apodization functions be conveniently generated with that approach but so can Laguerre-Gaussian (LG), and Bessel-Gaussian (BG) beams as long as we replace the original computer-generated phase hologram with a new one that carries information of both suitable phase and corresponding amplitude. Having known the different properties of different pupil apodization functions in the case of not only propagation but also tight focusing [30], the optical shaping generated from the LG, the Gaussian, and the BG beams are assumed to show some novel properties. Therefore, a new kind of phase modulated radially polarized beam called doughnut Gaussian (DG) beam is introduced in a high NA focusing system. The defined DG beam is similar to a hollow Gaussian beam. In 2010, a sub-wavelength focal spot was achieved by using a radially polarized narrow-width annular beam [31]. The DG beam, which is similar to the narrow-width annular beam, is falling to the category of the Gaussian beams [32]. Considering that the intensity is null at the center of the doughnut beam, the focusing field of a radially polarized DG incident beam through a high NA lens will exhibit high resolution. In this article, we demonstrate the focusing performance of a longitudinally polarized doughnut gaussian beam with high NA focusing system.

2. Principle of the optical focusing system

A schematic diagram of the suggested method is shown in Fig.1.The longitudinally polarized doughnut Gaussian beam is focused through a multi belt binary phase filter (MBBPF) and is subsequently focused by a high NA lens. The analysis was performed on the basis of Richards and

*Corresponding Author: prabakaran27mar@gmail.com Received: 19.02.2018 Accepted: 25.02.2018 Published on: 20.03.2018

> Prabakaran et al., .

RESEARCH ARTICLE

Wolf's vectorial diffraction method [33] widely used for high-NA lens system at arbitrary incident polarization. In the case of the incident polarization, adopting the cylindrical coordinates r,z, ϕ and the notations of Ref. [34], The focal field of a radially polarized doughnut Gaussian beam (RPDGB) can be written as

$$\vec{E}(r,z) = E_r \vec{e}_r + E_z \vec{e}_z \quad (1)$$

Where E_r and E_z are the amplitudes of the two orthogonal components \vec{e}_r and \vec{e}_z are their corresponding unit vectors.

The two orthogonal components of the electric field is given as

$$E_{r}(r,z) = A \int_{0}^{\alpha} \cos^{1/2}(\theta) T(\theta) \sin 2\theta J_{1}(kr\sin\theta) e^{ikz\cos\theta} d\theta \quad (2)$$
$$E_{z}(r,z) = 2iA \int_{0}^{\alpha} \cos^{1/2}(\theta) T(\theta) \sin^{2}\theta J_{0}(kr\sin\theta) e^{ikz\cos\theta} d\theta \quad (3)$$

Where $k = 2\pi/\lambda$ is the wave number and J_n(x) is the Bessel function of the first kind with order n. r and z are the radial and z coordinates of observation point in focal region, respectively. T(θ) describe the amplitude modulation. For a high NA lens, the electric field of the DG beam at the output pupil is defined as follows

$$T(\theta) = \exp\left[-\left(\frac{\sin(\theta) - \theta_0}{\omega_0}\right)^2\right]$$
(4)

where ω_0 reflects the beam size at the beam waist of the Gaussian beam. θ_0 relates with the radius of the DG beam. θ is the variable of the function. Obviously, the shape of the defined doughnut Gaussian beam is determined by θ_0 and ω_0 . To be more specific, the position of the maximum field intensity depends on θ_0 . For $\theta_0=0$ the beam governed by Eq. (4) is a conventional Gaussian beam. The width of the DG beam is determined by ω_0 [35]. The effect of four belt binary phase filter on the input radially polarized doughnut Gaussian beam is evaluated by replacing the function $T(\theta)$ by $T(\theta)BPF(\theta)$. Where $BPF(\theta)$ is given by

$$BPF(\theta) = \begin{cases} 1, for 0 < \theta < \theta_1, \theta_2 < \theta < \theta_3, \\ -1 for \theta_1 < \theta < \theta_2, \theta_3 < \theta < \theta \max \end{cases}$$
(5)

Specially designed optical element has set of four angles. The set of four angles are optimized by traditional globalsearch-optimization algorithm. Based on this algorithm we choose one structure with random values for θ_1 to θ_3 from all possibilities and simulate their focusing properties by vector diffraction theory. If the structure generates a sub wavelength single or multiple focal holes and satisfies the limiting conditions that that the FWHM of the generated focal spot segment is less than 0.5λ , it is chosen as the initial structure during the optimization procedures.



Fig.1. Schematic diagram of the proposed system radially polarized doughnut Gaussian beam passes through a multi belt binary phase filter and is subsequently focused by a high-NA lens

In the following steps, continue to vary θ of one chosen zone to generate a single or splitted focal spots on axial focal field until the generation of uniform intensity on axial profile without affecting the limiting condition. Basing on the above equations, focusing properties of radially polarized DGB with binary phase plate can be investigated theoretically.

3. RESULTS AND DISCUSSION

We performed the integration of Eq. (1) numerically for NA=0.95 and $\lambda = 1$, $\alpha = \arcsin(NA)$, $\theta_0 = 0.8$ for all calculation in the length unit is normalized to λ and the energy density is normalized to unity. In order to understand focusing properties of the phase modulated radially polarized doughnut Gaussian beam extensively, firstly, the focusing of radially polarized doughnut Gaussian beam with high NA lens system is investigated without phase



Fig.2. The contour plots of the plane radially polarized doughnut gaussian beam focused by high NA lens. The intensity profiles of the (a) radial, (b) azimuthal and (c) total intensity distribution components of the optical field at near the focus d) 2D intensity distribution for NA=0.95 and θ_0 =0.8.

modulation. Fig.2(a-c) illustrates the evolution of threedimensional intensity distribution of radial component, azimuthal component and total intensity of the high NA lens in r-z plane for incident radially polarized doughnut

Prabakaran et al.,

Gaussian beam (DGB). From Fig. 2d and 2c we measured the radial component is 20% of the total intensity and full width half maximum (FWHM) of the generated focal spot as 0.64 λ and its focal depth as 1.4 λ respectively. Such a

focal spot segment is useful for high refractive index particle trapping.

To realize a relatively long longitudinal polarized light needle with uniform intensity and a narrower spot size, a four belt binary phase optical element is introduced into the optical focusing system. We observed that it is possible to generated sub wavelength focal spot segment by properly tuning the phase of incident radially polarized DG beam using a four belt binary phase filter (BPF). We choose one structure with random values for θ_1 - θ_3 from all possibilities and simulate their focusing properties by vector diffraction theory. Fig. 3(a) shows the generation of large focal depth of single sub-wavelength longitudinal focal spot segment generated by the high NA lens for the MBBPF with θ_1 =16.56°, θ_2 =43.93°, θ_3 =62.28° θ_{max} =71.84° for $\theta_0 = 0.8$. It is observed from the Figure 3(a) shows the intensity profile of the radial, longitudinal and total electric field components of the optical field at focus.



Fig. 3 The contour plots of the phase modulated radially polarized doughnut Gaussian beam focused by high NA lens. The intensity profiles of the (a) radial, (b) azimuthal and (c) total intensity distribution components of the optical field at near the focal plane, (d) 2D intensity distribution for $\theta_0=0.8$.

It is evident that the intensity of the longitudinal component (blue line) is higher than the radial component (red line). From the Fig. 3d, measured the intensity distribution of radial component is only 11% of the normalized total intensity and hence the generated beam is strongly polarized in the longitudinal direction and the FWHM of the generated focal spot is 0.42λ and focal depth of 5λ which is shown in Fig.3(c). Such a needle of sub wavelength longitudinal polarized beam with long focal depth finds its application in near field optical recording and particle acceleration. The depth of focus (DOF) of the focal spot generated by our method extends over two times than that mentioned in recent papers [36-39]. This result is also better than other methods mentioned before. Table 1 shows the comparison of FWHM of the focal spot and its corresponding focal depth generated by the high NA lens system under different methods.

Table .1 Shows that the comparisons of spot size &	
depth focus for different methods	

Methods	Spot size	Depth of focus
Plane DG beam	0.64 λ	1.4 λ
Five belt binary optics method [36]	0.43λ	$\sim 4\lambda$
Annular beam method [37]	0.43λ	$\sim 4\lambda$
Four Belt binary optics method [38]	0.44λ	$\sim 4\lambda$
Three belt binary [39]	~0.44λ	~2λ
Proposed method	0.42 λ	5λ

Hence in order to achieve multiple trapping, we suggested a MBBPF optimized with angles are θ_1 =16.56°, θ_2 =43.93°, $\theta_3=62.28^\circ$ $\theta_{max}=71.84^\circ$. These angles are optimized based on the above-mentioned method but with the limiting condition that the FWHM of the generated focal spot segment is less than 0.5λ , and there should be at least two such focal spots in the focal segment. We observed from the Fig.4(d&c), it is possible to generated series of splitted focal spot each having FWHM of 0.72 λ and DOF is 1.8 λ and are axially separated by a distance of 4.4 λ . Further we measured the radial component is 40% of the total intensity is shown in Fig.4(d). The parameter need to obtain this profile are $\theta_0=0.7$. Such a focal splitting is useful in controlling the position of the optical traps. The above simulation calculations show that, by utilizing the BPF to modulate the phase of radially polarized doughnut gaussian beam (RPDGB), the optical spot in the focal region can be used as a powerful tool for particle manipulation [40-42].



Fig.4. Formation of an splitted focal spots in the *r*-*z* plane after a specially designed four-belt phase filter. (a), longitudinal component (b) radial component (c) The total intensity distribution. (d) 2D intensity distribution for θ_0 =0.7.

4. CONCLUSION

In conclusion, the proposed multi belt binary phase modulation scheme with high NA lens for the incident radially polarized doughnut Gaussian beam to generated a needle of longitudinally polarized beam with long focal depth (5λ) and sub wavelength focusing (0.42λ) is analyzed and demonstrated numerically by using vector diffraction theory. Apart from generating single focal spot with long depth of focus, we have also showed the possibility of generated splitted focal spot segments

Prabakaran et al.,

trough properly designed multi belt binary phase filter with radius of the DG beam. These kind of focal spots have been used to optical trapping, the material processing technologies, optical recording, and optical trapping of high refractive index particle.

REFERENCES

- [1]. Cicchitelli, L H.Hora R. Postle, 1990. Longitudinal field components for laser beams in vacuum. Phys. Rev. A 41, 3727.
- [2]. Biss, D.P., Brown, T.G, 2003. Polarization-vortexdriven second-harmonic generation. Opt. Lett. 28, 923-925.
- [3]. Novotny, L., Beversluis, M.R., Youngworth, K.S., Brown, T.G. 2001. Longitudinal Field Modes Probed by Single Molecules. Phys. Rev. Lett. 86 (2001) 5251
- [4]. Zhang,Y., Bai,J. 2009. Improving the recording ability of a near-field optical storage system by higher-order radially polarized beams. Opt. Express 17, 3698-3706.
- [5]. Zhan,Q.2004. Trapping metallic Rayleigh particles with radial polarization. Opt. Express 12, 3377– 3382.
- [6]. Quabis, S., Dorn,R.,Eberler, M.,Glockl,O., Leuch,G.2000. Focusing light to a tighter spot. Opt Commun 179. 1–7.
- [7]. Dorn,R., Quabis,S., Leuchs,G.2003. Shaper focus for a radially polarized light beam. Phys Rev Lett 91, 233901.
- [8]. Wang, H.F., Shi,L.P., Lukyanchuk,B., Sheppard,C., Chong, C.T. 2008. Creation of a needle of longitudinally polarized light in vacuum using binary optics. Nat Photon 2, 501–505.
- [9]. Kitamura,K., Sakai,K., Noda.S.2010. Sub-wavelength focal spot with long depth of focus generated by radially polarized, narrow-width annular beam. Opt.Express 18, 4518–4525.
- [10]. Kuang,C., Hao,X., Liu,X., Wang,T., Ku,Y. 2011. Formation of sub-half-wavelength focal spot with ultra long depth of focus. Opt Commun 284,1766– 1769.
- [11]. Kozawa,Y., Sato,S.2007. Sharper focal spot formed by higher-order radially polarized laser beams. J Opt Soc Am A 24, 1793–1798.
- [12]. Kozawa,Y., Sato,S.2012. Focusing of higher-order radially polarized Laguerre-Gaussian beam. J Opt Soc Am A 29, 2439–2443.
- [13]. Guo,H., Weng,X., Jiang,M., Zhao,Y., Sui,G., Hu,Q.2013. Tight focusing of a higher-order radially polarized beam transmitting through multi-zone binary phase pupil filters. Opt Express 21. 5363–5372.
- [14]. Sheppard,C., Mehta, S. 2012. Three-level filter for increased depth of focus and Bessel beam generation Opt. Express 20,27212-27221.
- [15]. Liu,L., Diaz,F., Wang,L., Loiseaux,B., Huignard,J., SheppardC.J.R., Chen, N. 2008. Super resolution along extended depth of focus with binary-phase filters for the Gaussian beam. J. Opt. Soc. Am. A.25, 2095-2101.

- [16]. Yun,M., Wang,M., Liu,L. 2005. Superresolution with annular binary phase filter in the 4Pi confocal system. J. Opt. A: Pure Appl. Opt.7, 640-644.
- [17]. Zhang,Y., Ye,X. 2007. Three-zone phase-only filter increasing the focal depth of optical storage systems with a solid immersion lens. Appl. Phys. B 86, 97-103.
- [18]. Yu,J., Zhou, C., Jia,W.2010. Transverse superresolution with extended depth of focus using binary phase filters for optical storage system. Opt. Commun. 283, 4171-4177.
- [19]. Pereira,S.F., van de Nes,A.S. 2004. Particle-swarm optimization of axially super resolving binary-phase diffractive optical elements. Opt. Commun. 234, 119-
- [20]. Wang,H., Shi,L., Yuan,G., Miao,X., Tan,W., Chong,T.2006. Nanofocusing of longitudinally polarized light using absorbance modulation. Appl. Phys. Lett. 89. 061103.
- [21]. Ravi,V., Suresh, P., Rajesh, K.B.2012. Z.Jaroszewicz, P.M.Anbarasan and T.V.S. Pillai, Generation of subwavelength longitudinal magnetic probe using high numerical aperture lens axicon and binary phase plate. J. Opt. 14,055704.
- [22]. Lu,Y., Xie,J., Ming,H.2002. Binary pure-phase filter optimized the optical distribution of solid immersion lens Opt. Commun. 215, 251.
- [23]. Sheppard,C.J.R. 2013. Pupil filters for generation of light sheets.Opt. Express 21, 6339-6345.
- [24]. Huang,K., Shi,P., Kang,X., Zhang,X., Li,Y.2010. Design of DOE for generating a needle of a strong longitudinally polarized field Opt. Lett.35, 965-967.
- [25]. Wang, X. L., Ding, J., Qin, J.Q., Chen, J., Fan, Y.X., Wang, H.T.2009. Configurable three-dimensional optical cage generated from cylindrical vector beams, Opt. Commun. 282, 3421–3425.
- [26]. Keller, J., Schönle, A., Hell, S.W.2007. Efficient fluorescence inhibition patterns for RESOLFT microscopy, Opt. Express 15,3361–3371.
- [27]. Kozawa,Y., Sato,S.2012. Focusing of higher-order radially polarized Laguerre-Gaussian beam, J. Opt. Soc. Am. A 29, 2439–2443.
- [28]. Khonina,S.N., Golub,I.2012. Enlightening darkness to diffraction limit and beyond: Comparison and optimization of different polarizations for dark spot generation, J. Opt. Soc. Am. A 29, 1470–1474.
- [29]. Wang, X.L., Ding,J.P., Ni,W.J., Guo,C.S., Wang,H.T.2007. Generation of arbitrary vector beams with a spatial light modulator and a common path interferometric arrangement, Opt. Lett. 32,3549–3551.
- [30]. Tovar,A.A.1998. Production and propagation of cylindrical polarized Laguerre-Gaussian beams, J. Opt. Soc. Am. A 15, 2705–2711.
- [31]. Kitamura,K., Sakai,K., Noda,S.2010. Sub-wavelength focal spot with long depth of focus generated by radially polarized, narrow-width annular beam. Opt Express 18, 4518–4525.
- [32]. Liu Zhengjun, Zhao Haifa, Liu Jianlong, Lin Jie, Ahmad Muhammad A, Liu Shutian.2007. Generation of hollow Gaussian beams by spatial filtering. Opt Lett 32, 2076–2078.

Prabakaran et al.,

- [33]. Richards,B., Wolf,E.1959. Electromagnetic diffraction in optical systems, II. Structure of the image field in an aplanatic system, Proc. R. Soc. Lond. A Math. Phys. Sci.253, 358–379.
- [34]. Youngworth,K.S., Brown,T.G.2000. Focusing of high numerical aperture Cylindrical vector beams, Opt. Express 7,77–87.
- [35]. Jie Lin., Ran Chen., Haichao Yu, Peng Jin., Michael Cada., Yuan Ma.2014. Analysis of sub-wavelength focusing generated by radially polarized doughnut Gaussian beam. Optics & Laser Technology 64, 242-246.
- [36]. Wang,H.F., Shi,L.P., Lukyanchuk,B., Sheppard,C., Chong, C.T. 2008.Creation of a needle Of longitudinally polarized light in vacuum using binary optics, Nat. Photonics 2, 501–505.
- [37]. Kitamura,K., Sakai, K., Noda,S.2010. Subwavelength focal spot with long depth of focus generated by radially polarized, narrow-width annular beam, Opt.Express 18, 4518–4525.
- [38]. Huang,K., Shi, P., Kang,X.L., Zhang,X., Li,Y.P.2010. Design of DOE for generating a needle of a strong longitudinally polarized field, Opt. Lett. 35, 965– 967.
- [39]. Zhongquan Nie, Guang Shi, Dongyu Li, Xueru Zhang, Yuxiao Wang, Yinglin Song.2015. Tight focusing of a radially polarized Laguerre–Bessel–Gaussian beam and its application to manipulation of two types of particles, Physics Letters A 379, 857-863.
- [40]. Prabakaran,K., Rajesh,K.B., Sumathira,S., Dhivya bharathi,M Hemamalini,R., Mohamed Musthafa,M., Aroulmoji,V.2016. Creation of multiple subwavelength focal spot segments using phase modulated radially polarized multi Gaussian beam" Chin. Phys. Lett. 33, 094203.
- [41]. Sundaram, C. M., Prabakaran, K., Anbarasan, P. M., Rajesh,K.B., Musthafa,A.M.2016. Creation of Super Long Transversely Polarized Optical Needle Using Azimuthally Polarized Multi Gaussian Beam, Chin. Phys. Lett. 33, 064203.
- [42]. Chandrasekaran,R., Prabakaran,K.,Rajesh,K.B.2016. Generation of multiple focal spot and focal hole segments using phase modulated cylindrically polarized annular multi-Gaussian beam. Opt. and quantum electronic 48,39-48.