RESEARCH ARTICLE

Focal Shifting of Radially Polarized Laguerre Bessel Gaussian Beam with Radial Cosine Phase Plate

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ABSTRACT: Focal shift of radially laguerre polarized Bessel Gaussian beams induced by cosine phase masks are investigated theoretically by vector diffraction theory. Results show that when the radially polarized laguerre Bessel Gaussian beam with cosine phase plate is focused, the focal pattern differs considerably with frequency parameter (C) in the cosine function term. Increasing the value of frequency parameter in the cosine part of the phase mask, focal shift may occur, simultaneously, the focal shift direction may change. Moreover, by altering frequency parameter of the phase mask will change the energy distributions of maximum intensity peak and other small intensity peaks.

KEYWORDS: Vector diffraction theory; Optical Trapping; Laguerre polarized Bessel Gaussian beams; High NA lens.

DOI: 10.29294/IJASE.4.4.2018.740-745

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1. INTRODUCTION

Recently the focal shift, which refers to the phenomena that the peak or the axial maximum intensity of the focused beam is shifted away from the geometrical focus [1-3], has attracted much attention for several decades [4-6]. In past few decades, the focal shift has been studied for various of beams, for example, Gaussian beams [3,5], flattened Gaussian beams [7], Gaussian Schell-mode beams [8] and so on. These beams have the transverse maximum intensity along with the optical axis after passing through the focusing system, therefore the position of axial maximal irradiance of the focused beam is usually viewed as the actual focal position [2,3], which is determined by the axial intensity distribution of the beam. The focal shifts may play a crucial role in achievement of super resolution and in optical trapping [9]. In optical trapping system there are two kinds of forces acting on the particle. One is the optical gradient force, which plays a crucial role in constructing optical trap and its intensity is proportional to the optical intensity gradient; the other kind of force is scattering force, which usually has complex forms because this kind of force is related to the properties of the trapped particles, and whose intensity is proportional to the optical intensity. Optical gradient force is necessary for optical trapping, and its direction is opposite to the optical gradient direction. Tracing the movement of the point of absolute maximum intensity along optical axis has attracted many researchers for several decades [10-12]. It was found that the point of absolute maximum intensity does not coincide with the geometrical focus but shifts along optical axis. This phenomenon is referred to as focal shift. Focal shifts have taken an important part in design and optimization of various optical systems and extremely important for image formation in defocused planes, for increasing the depth of focus and in automatic focusing [13]. More interestingly, the focal shift may be in continuous in certain optical focusing systems. It was found that the focal shift may be accompanied by an effective permutation of the focal point, and this effect is referred to as focal switch [14,15]. The polarization and phase wave front are very important characteristics to alter propagating and focusing properties of beams [16-20].In our knowledge, the focusing properties of radially polarized Laguerre Bessel gaussian beam with cosine phase wave front are not studied. In this paper the focal shift of the radially polarized Laguerre Bessel gaussian beam with radial cosine phase plate is investigated theoretically by vector diffraction theory.

2. VECTOR DIFFRACTION THEORY OF RADIALLY POLARIZED LAGUERRE BESSEL GAUSSIAN BEAM

The radially polarized laguerre Bessel Gaussian beam with cosine phase plate and then focused through a high NA lens system is shown in Figure1. The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method [21] widely used for high-NA lens system at arbitrary incident polarization. In the case of the incident polarization, adopting the cylindrical coordinates r,z,φ and the notations of Ref. [22], the electric field E(r,z,φ) in the vicinity of the focal region can be written as

$$\vec{E}(r,z)=E_r\vec{e}_r+E_z\vec{e}_z \rightarrow (1)$$

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Received: 15.02.2018 **Accepted:** 10.04.2018 **Published on**: 27.05.2018

Where E_r and E_z are the amplitudes of the two orthogonal components $\vec{e}_r$ and $\vec{e}_z$ are their corresponding unit vectors. The two orthogonal components of the electric field is given as

$$E_r(r,z) = A\int_0^{\alpha}\sqrt{\cos(\theta)}\sin 2\theta P(\theta) J_1(kr\sin\theta)e^{ikz\cos\theta}d\theta \rightarrow (2)$$

$$E_z(r,z) = 2iA\int_0^{\alpha}\sqrt{\cos(\theta)}\sin^2\theta P(\theta) J_0(kr\sin\theta)e^{ikz\cos\theta}d\theta \rightarrow (3)$$

Where $\alpha = \arcsin(NA)/n$ the maximal angle is determined by the numerical aperture of the objective lens, and n is the index of refraction between the lens and the sample. $k = 2\pi/\lambda$ is the wavenumber and $J_n(x)$ is the Bessel function of the first kind with order n. r and z are the radial and z coordinates of observation point in focal region, respectively. P(θ) describes the pupil apodization function. The intensity in the focal region is proportional to the square modulus of Eq. (1). In the system investigated in this article, the wavefront phase distribution is radial cosine function distribution, and can be written as

$$\phi = D.\pi\cdot\cos\left[\pi\cdot C\frac{\tan(\theta)}{\tan(\alpha)}\right] \rightarrow (4)$$

where C is the frequency parameter in cosine part of the wavefront phase distribution, C denotes the radial change frequency of the phase parameter. A property of these phase masks is that the focal separation and energy distribution between foci (equivalently the number of foci) can be independently controlled by the filter parameters. The axial distance between foci is a function of mask parameter D, while the energy distribution between foci can be tuned by varying the mask parameter C. Here we fixed D=1 and wish to analyze the energy distribution between foci. The reason for choosing this kind of radial cosine phase wavefront is that it is very simple and easy to carry out, for example this kind of phase distribution can be implemented by phase spatial light modulator could also provide interactive control or by pure phase plate manufactured by the lithographic method conveniently.. The electric field in the focal region can be written in the same form as Eq. (1); However, the two orthogonal components Er and Ez are different and should be expressed as

$$E_r(r,z) = A\int_0^{\alpha}\sqrt{\cos(\theta)}\exp\left[i\cdot\pi\cdot\cos\left[\pi\cdot C\frac{\tan(\theta)}{\tan(\alpha)}\right]\right]\sin 2\theta P(\theta) J_1(kr\sin\theta)e^{ikz\cos\theta}d\theta \rightarrow (5)$$

$$E_z(r,z) = 2iA\int_0^{\alpha}\sqrt{\cos(\theta)}\exp\left[i\cdot\pi\cdot\cos\left[\pi\cdot C\frac{\tan(\theta)}{\tan(\alpha)}\right]\right]\sin^2\theta P(\theta) J_0(kr\sin\theta)e^{ikz\cos\theta}d\theta \rightarrow (6)$$

The total intensity distribution in the focal region is proportional to the square modulus of Eq. (1), and the radial polarized component and longitudinal polarized component can be calculated according to Eqs. (5) and (6) respectively. The $l(\theta)$ describes the radially polarized LBG beam, this function is given by [23]

$$l(\theta) = \beta^2\frac{\sin\theta}{\sin^2\alpha}L_0^1\left[\left(\frac{2\beta^2\sin^2\theta}{\sin\alpha}\right)^2\right]J_1\left(2\beta\frac{\sin\theta}{\sin\alpha}\right)\times\exp\left(\frac{-\beta^2\sin^2\theta}{\sin^2\alpha}\right) \rightarrow (7)$$

Where, β is the parameter that denoted the ratio of pupil diameter to the beam diameter and L_P^1 is the generalized Laguerre polynomial.

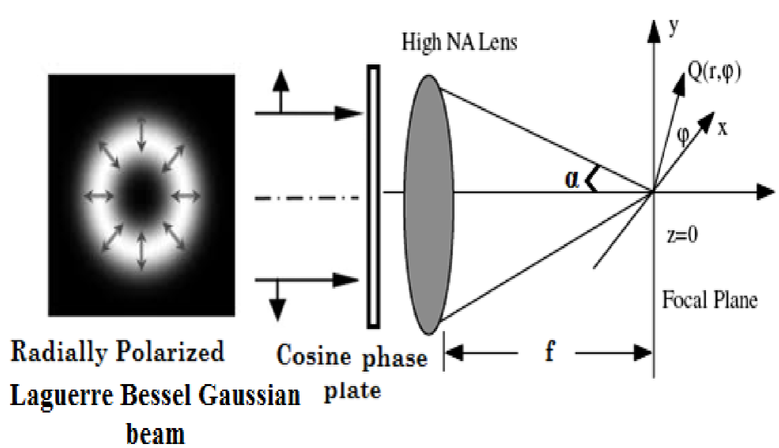

Fig.1.Focusing of a radially polarized Laguerre Bessel Gaussian beam with cosine phase plate by high NA lens

3. RESULTS AND DISCUSSION

Without loss of validity and generality, the focusing properties of the radially polarized Laguerre Bessel Gaussian beam with cosine phase wavefront are calculated. We perform the integration of Eq. (1) numerically using parameters λ = 1 and NA of the objective is 0.9. Here, for simplicity, we assume that the refractive index n =1 and A=1. For all calculation in the length unit is normalized to λ and the energy density is normalized to unity. In order to understand focusing properties of the radially polarized Laguerre Bessel Gaussian beam extensively, firstly, the focusing of radially polarized Laguerre Bessel Gaussian beam with high NA lens system is investigated without wave front phase modulation. Figure.2a shows the intensity profile of the radial, longitudinal and total electric field components of the optical field at focus. It is evident that the intensity of the longitudinal component (red line) is higher than the radial component (black line). From the Fig.2a, we measured the FWHM of the generated focal spot size is 0.56λ and radial component as 11% of the total intensity. The 3D intensity distribution near the focus as shown in Fig. 2c, reveals that the focal depth is 1.7λ. Such a focal spot segment is useful for high refractive index particle trapping. The Fig.3 illustrates the evolution of three-dimensional light intensity distribution of the high NA lens for the cosine parameter values of C=0.4, C=0.6 and C=5.2. It can be seen from Fig.3a–c the generated focal spot in focal region shifts towards the optical aperture along optical axis on increasing cosine parameter C, namely focal shift phenomenon occurs. Here focal shift denotes the movement of maximum intensity position. When increasing C to 5.2, the maximum light intensity peak stays near from optical aperture and focal depth is 2λ and is shown in Fig. 3.c & f. From this figure 4, we can see that the focal shift distance increases on increasing parameter C almost linearly and is illustrated by blue line in Fig.4. If this focal peak is used to construct one optical trap, cosine parameter C may be employed to adjust trap position, in turn, can transport micro particles. Further increasing the value of cosine parameter C is changed to investigate its effect on focal pattern evolution. The intensity distributions for C = 6.2 are illustrated in Fig.5. It can be seen from this figure, that the cosine parameter C affects focal intensity distribution very considerably. The focal spot segment is shifted to near from the optical aperture z=-10λ along the optical axis, so that there is only one optical intensity maximum and three minimum intensity peaks. From the figure.5 (a & b) we measure the FWHM of the focal spot size is 0.582 λ and corresponding focal depth is ~2 λ. We measure the radial component as 25% of the total intensity which is shown in fig.5 (a). However, the Fig.6 illustrates the evolution of three dimensional light intensity distribution of the high NA lens for the cosine parameter value of C = 6.4, It is observed multiple focal spot moves near and far from the optical aperture along optical axis which is shown in Fig.6(a) and (b). In optical trapping system, it is usually deemed that the forces exerted on the particles in light field include two kinds of forces, one is the gradient force, which is proportional to the intensity gradient; the other is the scattering force, which is proportional to the optical intensity. Therefore, the tunable focal shift predicts that the position of optical trap may be controllable and five peaks mean that there may appear five optical trappings. The Fig.7 shows the cosine parameter values of C=2.4, C=2.6, C=3 and C=6.8. It can be seen from Fig.7 a–c the generated focal spot in focal region shifts away from the optical aperture along optical axis on increasing cosine parameter C, namely focal shift phenomenon occurs in the positive on axial directions. When increasing C to 3.2, the maximum light intensity peak stays away from optical aperture is shown in Fig.7 (c & f). It is observed the focal shifting is almost linearly and is shown by the rose line in Fig. 8. If this focal peak is used to construct one optical trap, phase parameter C may be employed to adjust trap position, in turn, can transport micro particles. Further increasing the value of cosine parameter C = 11.6 are shown in Fig.9. It can be seen from this figure that the cosine parameter C affects the focal spot segment and is shifted far away from the optical aperture z=17λ along optical axis, from the figure we measure the FWHM of the focal spot size is 0.55 λ and corresponding focal depth is 2.5λ and is shown in fig.9 (a & b). From above focal pattern evolution process, we can see that for certain cosine parameter C can alter considerably intensity distribution in focal region of radially polarized Laguerre Bessel Gaussian beam and many novel focal patterns can occur, which can be used to construct tunable optical traps. Relative intensity value of the radially polarized component and longitudinal polarized component can affect the total intensity distribution considerably. Therefore, focusing properties of the radially polarized LBG beam may find very wide application in optical tweezers system and particle acceleration.

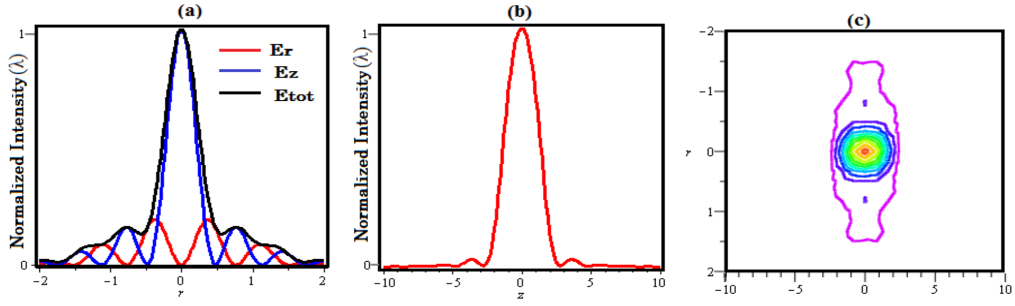

Fig.2. (a) 2D intensity distribution at z = 0λ. (b) On axial intensity at r= 0λ (c) 3D intensity distribution.

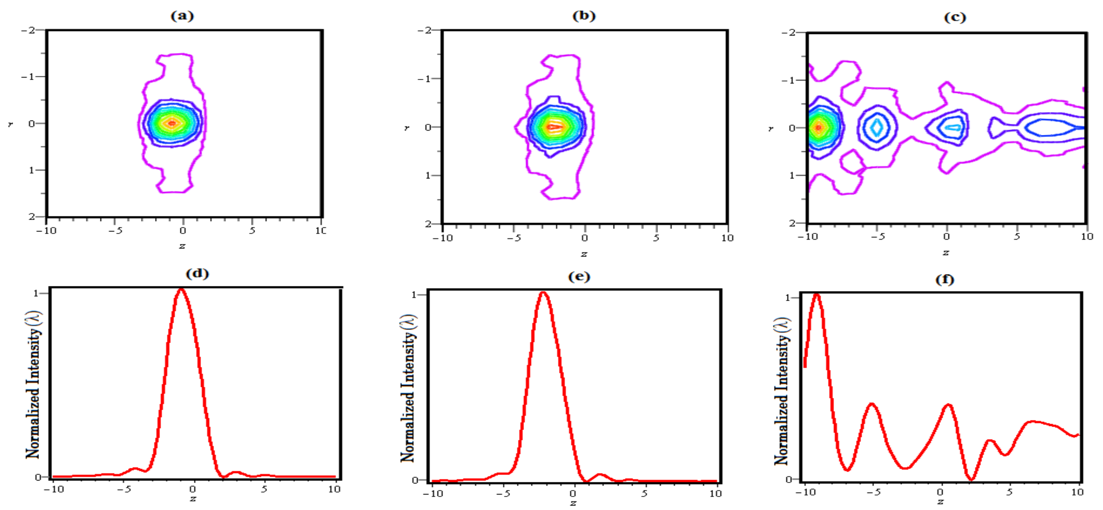

Fig.3. 3D and 2D Intensity distributions in focal region for NA=0.9, (a & d) C = 0.4, (b & e) C = 0.6, (c & f) C=5.2 respectively.

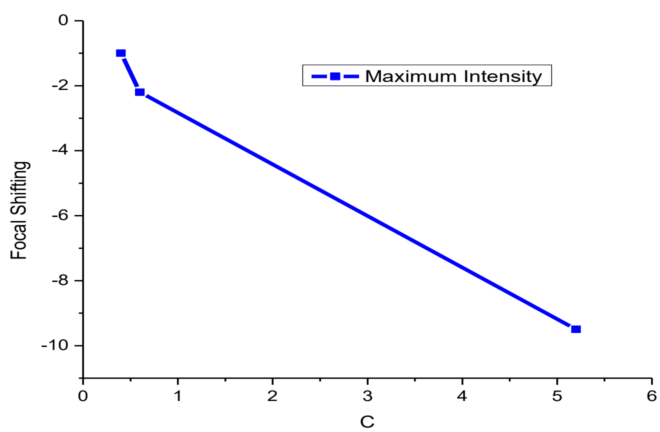

Fig.4. Dependence of focal hole shift near from the optical aperture under condition of NA = 0.9 on increasing C.

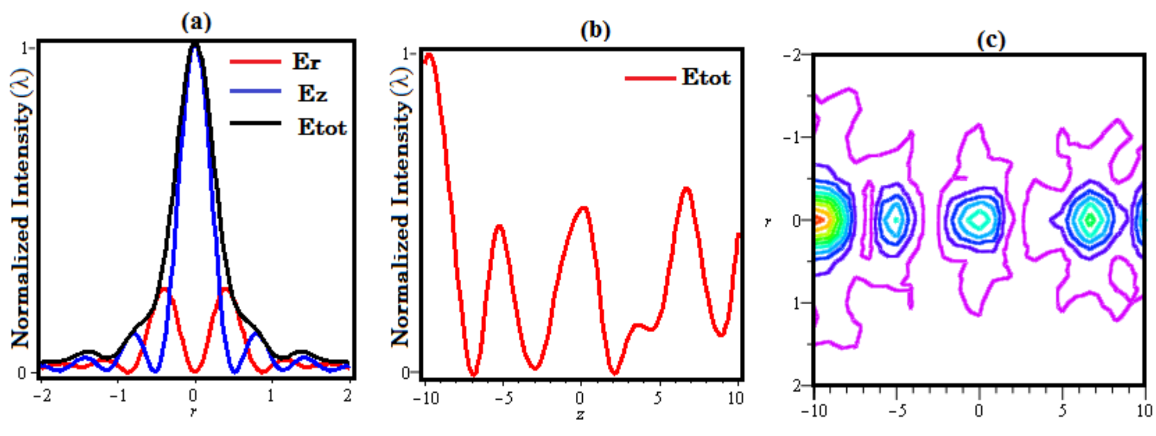

Fig.5. *(a) 2D intensity distribution at z = -10 λ. (b) On axial intensity at r= 0λ (c) 3D intensity distribution.*

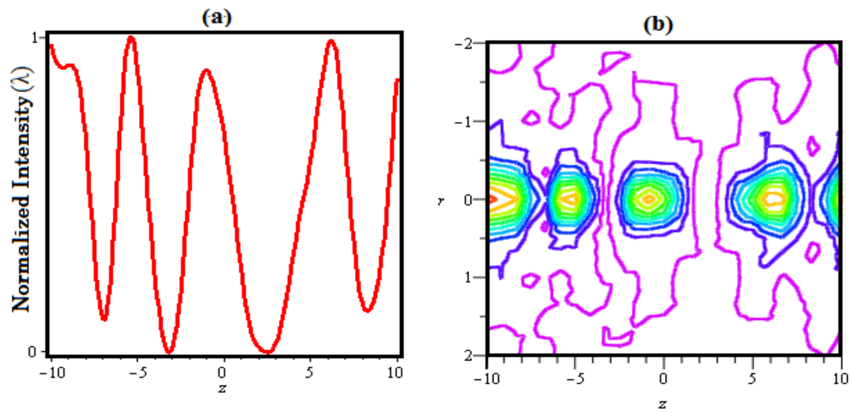

Fig.6. (a and b) 2D and 3D intensity distributions for C=6.4.

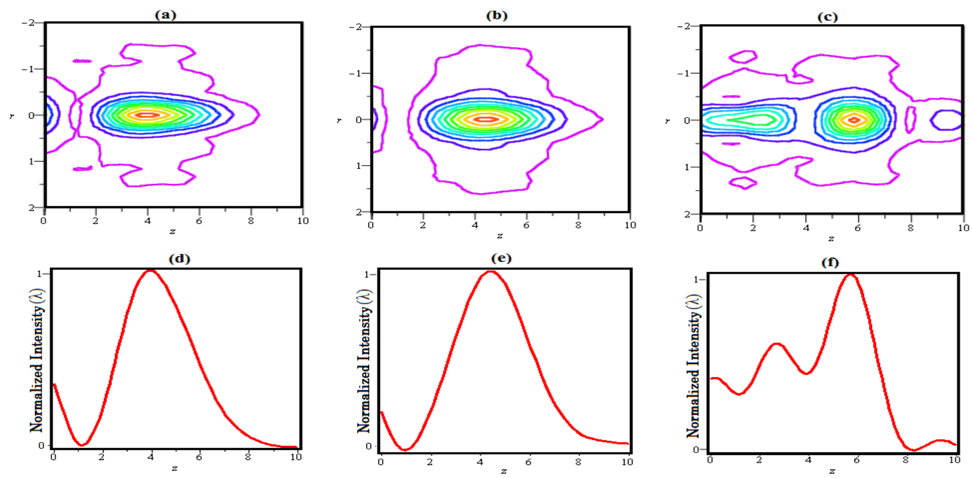

Fig.7. 3D and 2D Intensity distributions for NA=0.9, (a & d) C = 2.2, (b & e) C = 2.6 and (c & f) C = 3.2 respectively.

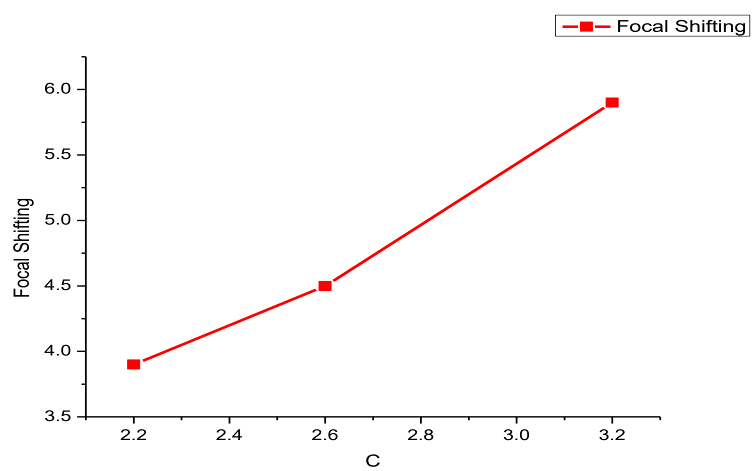

Fig.8. Dependence of focal hole shift far from the optical aperture under condition of NA = 0.9 on increasing C.

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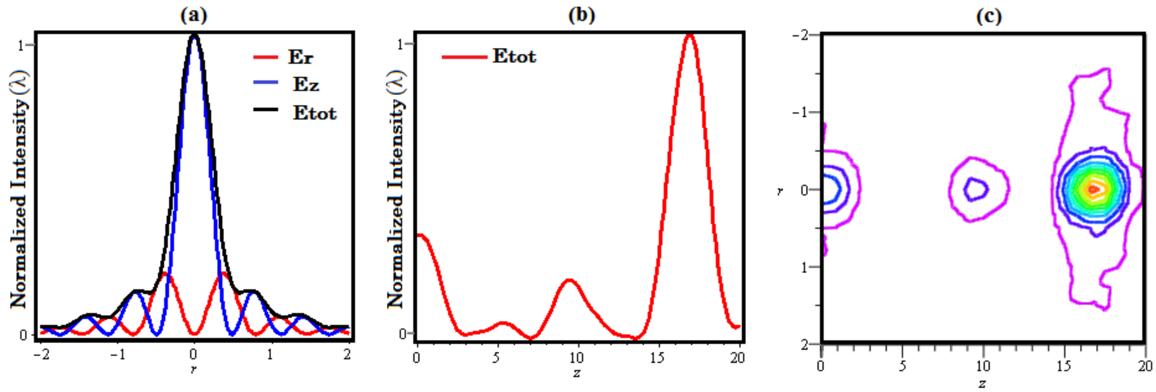

Fig.9*(a) 2D intensity distribution at z = 17 λ. (b) On axial intensity at r= 0λ (c) 3D intensity distribution.*

4. CONCLUSION

In conclusion, focal shift of radially polarized laguerre bessel gaussian beam investigated theoretically by vector diffraction theory in high numerical aperture lens system. Results show that the intensity distribution in focal region of the radially polarized laguerre bessel gaussian can be adjusted considerably by the cosine parameter C. Focus can shift along optical axis on increasing C, and focal pattern changes are observed. Moreover, by altering frequency parameter of the phase mask will change the energy distributions of maximum intensity peak and other small intensity peaks appeared. And novel focal patterns also evolve considerably, such a tunable focal shift can be used to construct and alterable optical trapping.

REFERENCES

[1]. Arimoto A, 1976. Intensity distribution of aberration-free diffraction patterns due to circular apertures F-number optical systems. Opt Acta.23, 245–250.

[2]. Li Y, Wolf E, 1981. Focal shifts in diffracted converging spherical waves. Opt Commun.39, 211–215.

[3]. Li Y, Wolf E. Focal shift in focused truncated Gaussian beams,1982. Opt Commun.42, 151–156.

[4]. Mahajan VN, 1983. Axial irradiance and optimum focusing of laser beams. Appl Opt. 22, 3042–3053.

[5]. Carter WH 1983. Focal shift and concept of effective Fresnel number for a Gaussian laser beam. Appl Opt. 21, 1989–1994.

[6]. Li Y, 1982. Dependence of the focal shift on Fresnel number and f number. J Opt Soc Am A 72, 770–775.

[7]. Ji X, Lu. B, 2003. Focal shift and focal switch of flattened Gaussian beam in passage through an aperture bifocal lens. IEEE J Quantum Electron.3 9, 172–178 .

[8]. Wang L, Lu B, 2003. Focal shift of partially polarized Gaussian Schell model beams. Optik. 114, 169–174.

[9]. Wang J, Liu L, Yun M, Liu D, Liu X,2005. Super resolution, extended focal depth or focal shift with electrically controllable radial birefringent filter. J. Opt. A: Pure Appl. Opt. 7, 748.

[10]. MacDonald M.P, Paterson L, Sibbett W, Dholakia K., Bryant P.E,2001. Trapping and manipulation of low-index particles in a two-dimensional interferometric optical trap. Opt. Lett. 26, 863–865.

[11]. Li Y, Wolf E,1981. Focal shifts in diffracted converging spherical waves, Opt. Commun.39, 211–215.

[12]. Gao X, Zhou F, Xu W, Gan F,2005. Gradient force pattern, focal shift, and focal switch in an apodized optical system. Optik.116, 99–106.

[13]. Odeja-Castaneda J, Berriel-Valdos L R,1988. Arbitrarily high focal depth with finite apertures, Opt. Lett. 13, 183–185.

[14]. Martines-Corral M, Climent V,1996. Focal switch: a new effect in low-Fresnel number system, Appl. Opt. 35, 24–30.

[15]. Li Y,1997. Focal shift and focal switch in dual-focus system. J. Opt. Soc. Am. A 14, 1297–1300 .

[16]. Gao X, Zhou F, Zhang F, Yang J T, Xu W, Gan F, 2005. Tunable focal depth of an apodized focusing system. Opt. Eng. 44, 063001.

[17]. Zhan Q and Leger J.R,2002. Focus shaping using cylindrical vector beams. Opt. Exp. 7, 324-331.

[18]. Xiumin Gao, Song Hu, Haitao Gu, Jian Wang,2006. Focal shift of three-portion concentric piecewise cylindrical vector beam. Optik 120, 519–523.

[19]. Gao X, Li J, Li Z, Wang J, Zhuang S,2010. Focusing of radially polarized beam with radial cosine phase wavefront. Optik 121, 1674–1680.

[20]. Xiumin Gao, Zhou Fei, Wendong Xu, Fuxi Gan,2004. Focus splitting induced by a pure phase-shifting apodizer. Optics Communications. 239, 55–59.

[21]. Richards B, Wolf E,1959. Electromagnetic diffraction in optical systems, II. Structure of the image field in an aplanatic system. Proc. R. Soc. A, Math. Phys. Eng.Sci.253 (1274) 358–379.

[22]. Youngworth, K.S, Brown TG, 2000. Focusing of high numerical aperture cylindrical vector beams. Opt. Express 7, 77–87.

[23]. Zhongquan Nie, Guang Shi, Dongyu Li, Xueru Zhang, Yuxiao Wang, Yinglin Song ,2015.Tight focusing of a radially polarized Laguerre–Bessel–Gaussian beam and its application to manipulation of two types of particles. Physics letter A 379, 857-863.